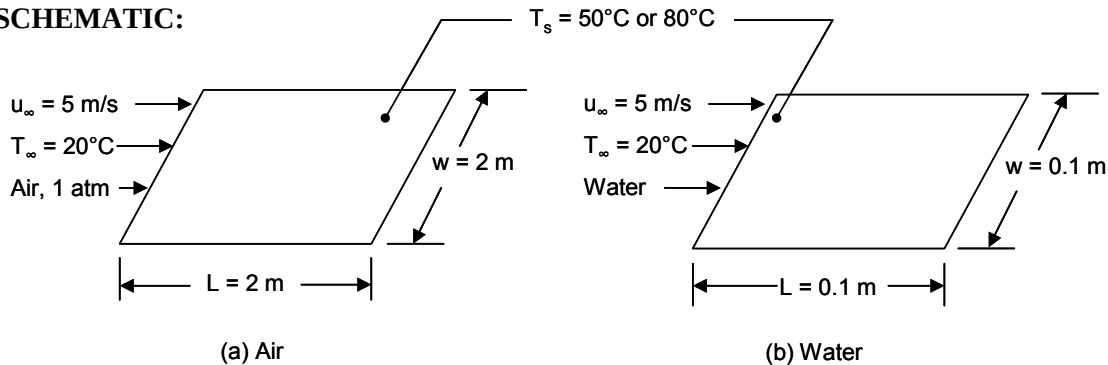


PROBLEM 7.13

KNOWN: Dimensions and surface temperatures of a flat plate. Velocity and temperature of air and water flow parallel to the plate.

FIND: (a) Average convective heat transfer coefficient, convective heat transfer rate, and drag force when $L = 2$ m, $w = 2$ m. (b) Average convective heat transfer coefficient, convective heat transfer rate, and drag force when $L = 0.1$ m, $w = 0.1$ m.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Boundary layer assumptions are valid, (3) Constant properties, (4) Transition Reynolds number is 5×10^5 .

PROPERTIES: Using *IHT*, Air ($p = 1$ atm, $T_f = 35^\circ\text{C} = 308$ K): $\text{Pr} = 0.706$, $k = 26.9 \times 10^{-3}$ W/m·K, $\nu = 1.669 \times 10^{-5}$ m²/s, $\rho = 1.135$ kg/m³. Air ($p = 1$ atm, $T_f = 50^\circ\text{C} = 323$ K): $\text{Pr} = 0.704$, $k = 28.0 \times 10^{-3}$ W/m·K, $\nu = 1.82 \times 10^{-5}$ m²/s, $\rho = 1.085$ kg/m³. Water ($T_f = 308$ K): $\text{Pr} = 4.85$, $k = 0.625$ W/m·K, $\nu = 7.291 \times 10^{-7}$ m²/s, $\rho = 994$ kg/m³. Water ($T_f = 323$ K): $\text{Pr} = 3.56$, $k = 0.643$ W/m·K, $\nu = 5.543 \times 10^{-7}$ m²/s, $\rho = 988$ kg/m³.

ANALYSIS:

(a) We begin by calculating the Reynolds numbers for the two different surface temperatures:

$$\text{Re}_{L1} = \frac{u_\infty L}{\nu_1} = \frac{5 \text{ m/s} \times 2 \text{ m}}{1.669 \times 10^{-5} \text{ m}^2/\text{s}} = 5.99 \times 10^5$$

$$\text{Re}_{L2} = \frac{u_\infty L}{\nu_2} = \frac{5 \text{ m/s} \times 2 \text{ m}}{1.82 \times 10^{-5} \text{ m}^2/\text{s}} = 5.49 \times 10^5$$

Therefore, in both cases the flow is turbulent at the end of the plate and the conditions in the boundary layer are “mixed.”

The average drag coefficient can be calculated from Equation 7.40. For the first case,

$$\begin{aligned} \bar{C}_{f,L1} &= 0.074 \text{Re}_{L1}^{-1/5} - 1742 \text{Re}_{L1}^{-1} \\ &= 0.074(5.99 \times 10^5)^{-1/5} - 1742(5.99 \times 10^5)^{-1} = 2.27 \times 10^{-3} \end{aligned}$$

Then

$$\begin{aligned} F_{D1} &= \bar{C}_{f,L1} \frac{1}{2} \rho u_\infty^2 A_s \\ &= 2.27 \times 10^{-3} \times \frac{1}{2} \times 1.135 \text{ kg/m}^3 \times (5 \text{ m/s})^2 \times 8 \text{ m}^2 = 0.257 \text{ N} \\ &= 0.257 \text{ N} \end{aligned}$$

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Continued...

PROBLEM 7.13 (Cont.)

The average Nusselt number is calculated from Equation 7.38, with $A = 871$ for a transition Reynolds number of 5×10^5 .

$$\begin{aligned}\overline{Nu}_{L1} &= (0.037 Re_L^{4/5} - 871) Pr^{1/3} \\ &= \left[0.037(5.99 \times 10^5)^{4/5} - 871 \right] (0.706)^{1/3} = 604.\end{aligned}$$

Then

$$\bar{h}_{L1} = \overline{Nu}_{L1} k / L = 604 \times 26.9 \times 10^{-3} \text{ W/m} \cdot \text{K} / 2 \text{ m} = 8.13 \text{ W/m}^2 \cdot \text{K} \quad <$$

and

$$q_1 = \bar{h}_{L1} A_s (T_s - T_\infty) = 8.13 \text{ W/m}^2 \cdot \text{K} \times 8 \text{ m}^2 \times (50^\circ\text{C} - 20^\circ\text{C}) = 1950 \text{ W} \quad <$$

Similarly for $T_s = 80^\circ\text{C}$ we find

$$F_{D2} = 0.227 \text{ N}, \bar{h}_{L2} = 7.16 \text{ W/m}^2 \cdot \text{K}, q_2 = 3440 \text{ W} \quad <$$

(b) Repeating the calculations for water

$$Re_{L1} = \frac{u_\infty L}{\nu} = \frac{5 \text{ m/s} \times 0.1 \text{ m}}{7.291 \times 10^{-7} \text{ m}^2/\text{s}} = 6.86 \times 10^5$$

$$Re_{L2} = 9.02 \times 10^5$$

The flow is turbulent at the end of the plate in both cases.

$$\bar{C}_{f,L1} = 0.074(6.86 \times 10^5)^{-1/5} - 1742(6.86 \times 10^5)^{-1} = 2.49 \times 10^{-3}$$

$$F_{D1} = 2.49 \times 10^{-3} \times 1/2 \times 994 \text{ kg/m}^3 \times (5 \text{ m/s})^2 \times 0.02 \text{ m}^2 = 0.620 \text{ N} \quad <$$

$$\overline{Nu}_L = \left[0.037(6.86 \times 10^5)^{4/5} - 871 \right] (4.85)^{1/3} = 1450$$

$$\bar{h}_{L1} = 1450 \times 0.625 \text{ W/m} \cdot \text{K} / 0.1 \text{ m} = 9050 \text{ W/m}^2 \cdot \text{K} \quad <$$

$$q_1 = 9050 \text{ W/m}^2 \cdot \text{K} \times 0.02 \text{ m}^2 \times (50^\circ\text{C} - 20^\circ\text{C}) = 5430 \text{ W} \quad <$$

For the higher surface temperature,

$$F_{D2} = 0.700 \text{ N}, \bar{h}_{L2} = 12,600 \text{ W/m}^2 \cdot \text{K}, q_2 = 15,100 \text{ W} \quad <$$

COMMENTS: (1) For air, kinematic viscosity increases with increasing temperature. This decreases the Reynolds number which causes the transition to turbulence to move downstream, thereby decreasing the drag force and average heat transfer coefficient. The heat transfer rate increases for the higher surface temperature, however, because of the greater temperature difference between the surface and air. (2) For water, kinematic viscosity decreases with increasing temperature, causing the opposite trends as for air. The heat transfer rate increases dramatically for the higher surface temperature because of the increases in both the heat transfer coefficient and temperature difference. (3) Even though the water flows over a plate that is 400 times smaller, the drag force and heat transfer rate are larger than for air because of the smaller viscosity and greater density, thermal conductivity, and Prandtl number. The discrepancy is particularly great for the heat transfer rate. (4) The problem highlights the importance of carefully accounting for the temperature dependence of thermal properties.